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Use of Approximating Polynomials to Estimate Profiles of Wind, Divergence, and Vertical Motion

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ABSTRACT—"Least-squares" approximating polynomials are used to suppress bias and random errors in estimating vertical profiles of winds, divergence, and vertical motion. A quadratic polynomial is used to filter each wind profile. Profiles of divergence and vertical motion computed from a linear, a cross-product, and a quadratic two-dimensional (horizontal) approximating polynomial model and from the Bellamy technique are compared. The random-error variance component of the wind observations is estimated from the filtering polynomial prediction errors. In turn, the random-error variance component of the filtered wind, divergence, and vertical motion is determined from the wind observational error variance for the various models.

In the presence of nonlinear variation in the horizontal wind field, the Bellamy modeling assumption of linear wind variation introduces biased divergence errors. The bias divergence errors will persist through a considerable portion of the troposphere as a result of the thermal wind relation and, in the vertical integration, will cause large "spurious" vertical motion estimates of ω at the top of the profile. Divergence estimates from both the cross-product and the quadratic approximating polynomial models of the horizontal wind field tend to be less biased in this situation and normally produce superior vertical motion profiles.

1. INTRODUCTION

In the past 2 decades, many attempts have been made to evaluate the large-scale fields of divergence and vertical motion. The methods most widely used have been the adiabatic method, the kinematic method, and several methods employing vorticity. Previous studies (Palmén 1956, Endlich and Clark 1963, Eddy 1964) indicated that reliability of the kinematic technique was sufficient for use in diagnostic studies if some added means were employed to reduce the influence of observational errors and small-scale effects. The intent of our study is to develop improved diagnostic techniques for atmospheric energy studies by using filtering polynomials. This is accomplished by incorporating vertically filtered profiles of wind at each station to estimate a horizontally filtered profile of divergence. Then vertical motion is computed from the divergence by "least-squares" polynominal integration. Through these techniques, the effects of random errors are suppressed and only the variations associated with the scale of the primary and secondary circulation are retained.

2. THE FILTERING POLYNOMIAL TECHNIQUE

The diagnostic model is based on an application of least-squares approximating polynomials (Hildebrand 1956) that was initiated by Panofsky (1949) for the purpose of objective analysis and further applied by Gilchrist and Cressman (1954), Johnson (1957), and others. This approach assures that, if least-squares estimates are unbiased, they are better estimates of the true wind structure than basic observations because a portion of the random-error component is filtered. In addition, if the random errors are also independent, the polynomial estimates are minimum variance estimates (Kendall and Stuart 1958, 1961).

In the filtering of each wind profile, a second degree polynomial is fitted to seven adjacent observations of a wind component, thus leaving four degrees of freedom for the random-error component. The filtered estimate is made for the middle observation of the seven. The next filtered estimate is made by adding the next adjacent observation in height, deleting the lowest observation in height from the prior seven points, and predicting for the new middle observation. Only midpoint polynomial estimates are used because the random-error variance of this estimate is a minimum (Johnson 1965). Through this stepping procedure, profiles of filtered wind components are obtained.

The horizontally filtered divergence estimates are made by selecting from three to eight station wind profiles to estimate one profile of divergence near the center of the station array. The predicted eastward wind component by the quadratic model (the highest order approximating polynomial employed in the study) is

$$\hat{u}=A_0+A_1x+A_2y+A_{12}xy+A_{11}x^2+A_{22}y^2 \qquad (1)$$

where the A_is are the least-squares polynomial coefficients estimated from the wind component data for each isobaric level, and x and y are scaled longitudinal and latitudinal distances. The relation between x and longitude, λ, is

$$x=\frac{\lambda-(\lambda_{max}+\lambda_{min})}{1/2(\lambda_{max}-\lambda_{min})} \qquad (-1\leq x\leq +1) \qquad (2)$$

where λ_{max} and λ_{min} are the two extreme longitudes of all stations selected for estimation of a profile of divergence. A similar relation exists between y and latitude, ϕ. The square area circumscribed by λ_{max}, λ_{min}, ϕ_{max}, and ϕ_{min} is called the local region because all stations used for the determination of the approximating polynomial are located within this region. The local region may change as new stations are added or a different combination of stations is utilized.

In the experimental results, profiles of divergence and vertical motion will also be presented for linear and cross-product models of polynomial approximation based on a truncation of eq (1). For the linear model, only the first three coefficients are estimated; for the cross-product model, the first four coefficients are estimated. Because the approximations are by least squares, the only restriction on the number of stations employed is that the number must be equal to or exceed the number of coefficients in each model.

When the number of stations is equal to the number of coefficients, the normal equations of the least-squares method reduce to a set of ordinary simultaneous equations. In the situation where only three stations are employed for the linear model, estimates of divergence from the polynomial approximation will be identical to estimates by the Bellamy method (Bellamy 1949).

The divergence is estimated by

$$\nabla \cdot \mathbf{U}=\frac{1}{R}\left(\sec \phi \frac{\partial u}{\partial \lambda}+\frac{\partial v}{\partial \phi}-v \tan \phi\right) \qquad (3)$$

when ∇ is the isobaric del operator, $\mathbf{U}$ is the velocity, R is the earth's radius, and v is the northward wind component. At the midpoint of the scaled region, the filtered divergence estimate in terms of the polynomial coefficients reduces to

$$\nabla \cdot \mathbf{U}=\left(\frac{A_1 \sec \phi}{\lambda_{max}-\lambda_{min}}+\frac{B_2}{\phi_{max}-\phi_{min}}-B_0 \tan \phi\right)/R \qquad (4)$$

where B_0 and B_2 are from the set of coefficients used in representing the northward component by a polynomial expression similar to eq (1).

The vertical motion, ω, at the dth isobaric level is

$$\omega=\omega(d-1)+\Delta\omega(d) \qquad (5)$$

where the incremental vertical motion from the isobaric equation of continuity is

$$\Delta\omega(d)=-\int_{p_{d-1}}^{p_d} \nabla \cdot \mathbf{U} dp. \qquad (6)$$

In evaluating this integral, a second-degree, least-squares polynomial was used to approximate the profile of filtered divergence over a vertical distance spanned by seven adjacent points. Then the segment of the polynomial between the midpoint observation and the adjacent lower point is used to represent the divergence between the limits in eq (6), and the integral is evaluated analytically. The entire profile of vertical motion is estimated by stepping through the divergence profile by the same manner utilized in computing the filtered wind profiles. This "ad hoc" technique of polynomial integration (Johnson 1965) has the advantage of combining polynomial filtering and integration. A more detailed derivation of this technique of integration and of the filtering technique for the wind and divergence profiles is presented by Schmidt and Johnson (1969).

3. EXPERIMENTAL RESULTS

The results obtained with the least-squares techniques are presented in three sections. Filtered profiles of wind are discussed in the first section. In the second section, profiles of divergence determined by the Bellamy technique using vertically filtered winds and profiles of vertical motion from the polynomial integration are presented. In the third section, the results from higher order approximating polynomials are compared.

The rawinsonde wind data obtained from the National Climatic Center, Asheville, N.C., were listed at 50-mb intervals from the surface to 200 mb. Above this level, the data interval decreased. In this study, we shall be unable to analyze the validity of two assumptions utilized in the data reduction. These assumptions are that a balloon's trajectory may be used to estimate the wind and that discrete data at 50-mb intervals provide representative observations of a true, large-scale wind profile.

The first assumption should be valid because the balloon acts as an integrator of small-scale wind fluctuations and the relative, large-scale motion of the air past the balloon may be neglected (Perkins 1952). The validity of the second assumption appears questionable from Reiter's example (1963, fig, 2.122.1, p. 26) showing systematic departure of the coded wind message from the profile determined from original rawin ascent records and from the work of De Mandel and Scoggins (1967). However, for the scale for which horizontal divergence is determined in this study, the 50-mb vertical data interval should be sufficient.

Filtered Winds

In figure 1, both the basic data (open circles) and the filtered estimates (solid line) for the eastward wind component are presented for Montgomery, Ala. The scatter of the open circles about the smooth profile is consistent with the suggestion by least-squares theory that the random and small-scale components will be removed. Note the excellent description of the 65-m/s jet maximum.

An added advantage of using approximating polynomials is that estimates of random-error variance may be determined from the residual sums of squares provided that observational errors are random and independent and the approximating polynomial is unbiased.

In figure 1, a one-standard-deviation confidence interval is denoted by dotted lines. If this confidence interval were plotted about the true profile and the random errors were

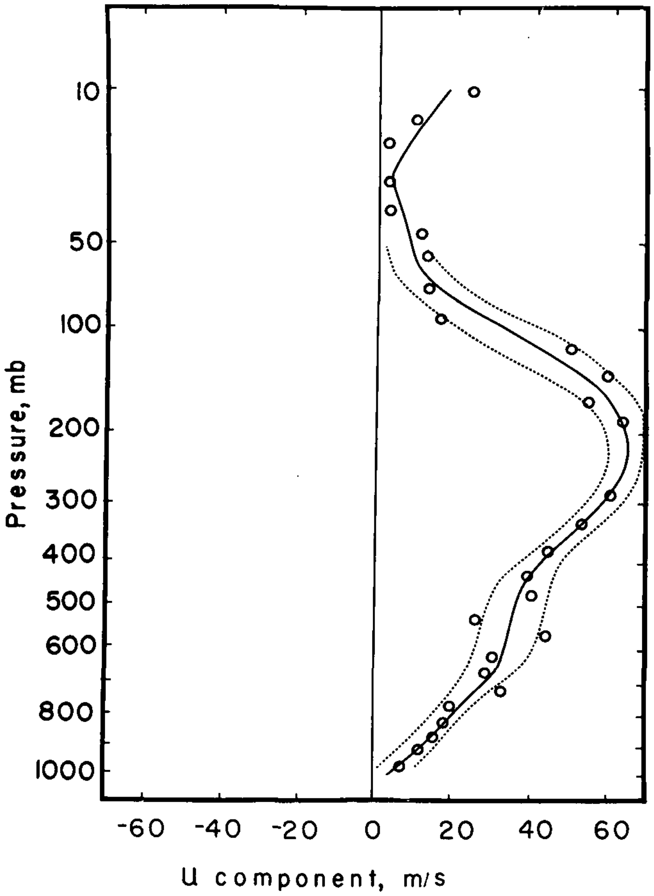

FIGURE 1.—Filtered eastward wind component profile for Montgomery, Ala., on 0000 GMT, Dec. 12, 1960. Open circles indicate wind observations. Dotted curves represent a one-standard-deviation confidence interval.

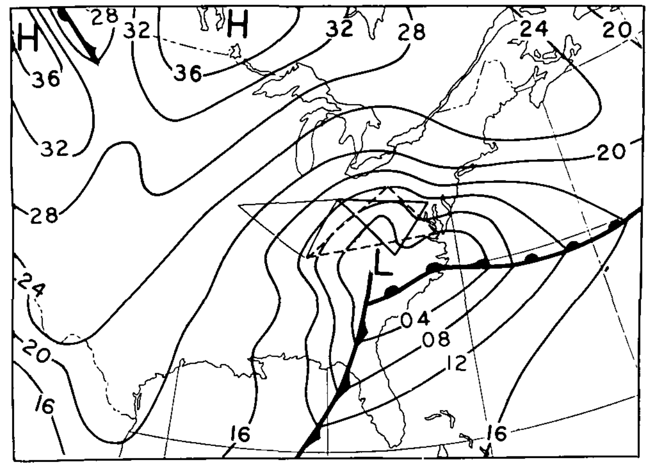

FIGURE 2.—Sea-level pressure distribution for 0000 GMT, Dec. 12, 1960. The three triangles designate areas for which divergence and vertical motion are estimated.

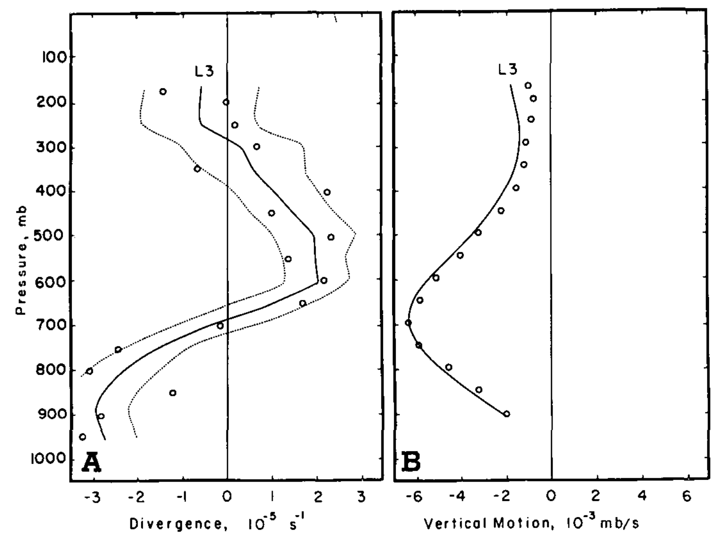

FIGURE 3.—Filtered (A) divergence and (B) vertical-motion profiles computed by the Bellamy technique (L3) for a triangle bounded by Columbia, Mo.; Dayton, Ohio; and Nashville, Tenn., for 0000 GMT, Dec. 12, 1960. Circles in (A) and (B) are unfiltered estimates of divergence and vertical motion, respectively. Dotted lines represent a one-standard-deviation confidence interval.

normally distributed, the interval should contain two-thirds of the wind observations. In the 1000- to 50-mb interval, roughly two-thirds of the observations are within the limits. The variance of the filtered midpoint estimate from the seven-point filtering interval for the quadratic approximation will be 0.333 times the observational variance. Thus, the confidence interval for filtered wind estimates would be slightly less than 0.6 of the interval shown in figure 1.

Divergence and Vertical Motion Estimates—Bellamy Technique

To check the consistency of the Bellamy technique for estimating divergence, we computed vertical motions in overlapping triangles. Typical examples from 0000 GMT on Dec. 12, 1960, are presented for the surface synoptic situation shown in figure 2. In figure 3A, the divergence, calculated by the Bellamy technique (the linear polynomial) for the triangle bounded by Columbia, Mo., Dayton, Ohio, and Nashville, Tenn. (light, solid line on fig. 2), shows excellent agreement with the synoptic situation. The convergence that occurs below 775 mb and the upper level divergence display Dines' compensation. The profile is similar to the profile of divergence for a developing cylcone presented by Petterssen (1956).

In figure 3B, the filtered vertical motion profile (solid line) with an extremum of -6.4×10^{-3} mb/s occurring at 700 mb agrees with the results of Danielsen (1966) and Krishnamurti (1968). The estimate of ω also approaches a small value in the stratosphere. The circles in figure 3B are vertical motion estimates from the polynomial integration of unfiltered divergence estimates. The differences are small and indicate that, if only vertical motion estimates are desired, the preliminary filtering is unnecessary.

The dotted curves in figure 3A bound the 68-percent (one standard deviation) confidence interval for the

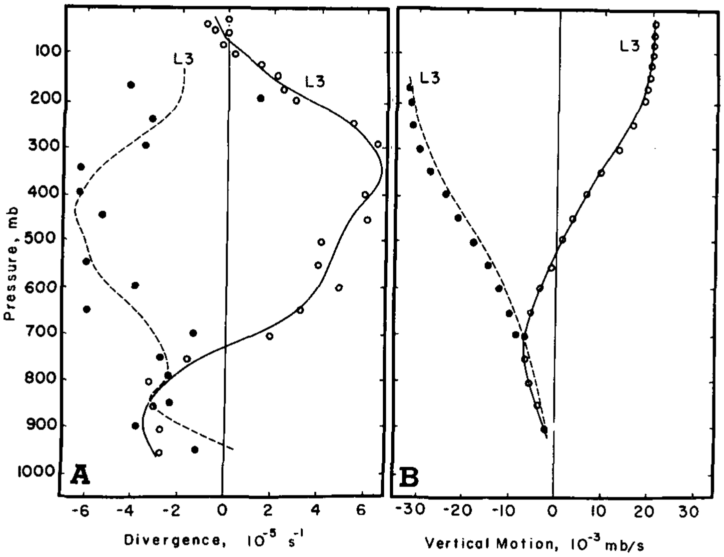

FIGURE 4.—Filtered (A) divergence and (B) vertical motion for two overlapping triangles for 0000 GMT, Dec. 12, 1960. Solid curves—Dayton; Washington, D.C.; and Greensboro, N.C., triangle; dashed curves—Nashville; Pittsburgh, Pa.; and Norfolk, Va., triangle. Circles portray unfiltered estimates.

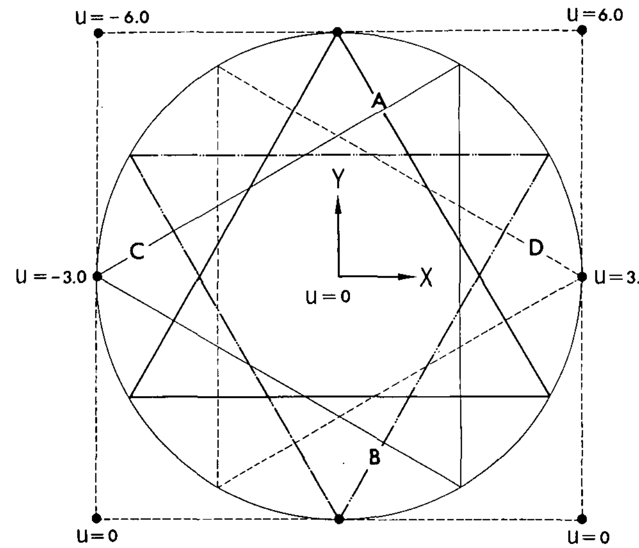

FIGURE 5.—Hypothetical distribution of four triangles to estimate divergence by the Bellamy method in a nonlinear wind field.

unfiltered divergence estimates. They include 12 of 17 points and indicate validity for standard deviation estimates that are determined through a variance analysis of the residual sums of squares. A corresponding confidence interval for filtered divergence, determined from the filtering weights, is 0.577 times the indicated interval. This supports use of polynomial filtering because the range is considerably reduced.

In some cases, the results were contradictory. The divergence and vertical motion profiles in figure 4 show conflicting results for the overlapped triangles shown in figure 2. The solid curves portray divergence and vertical motion estimates for the triangle bounded by Dayton, Ohio; Washington, D.C.; and Greensboro, N.C. (heavy, solid line); while the dashed curves portray estimates for the triangle bounded by Nashville, Tenn.; Pittsburgh, Pa.; and Norfolk, Va. (dashed line). Because the centroids of these triangles nearly coincide, the divergence and vertical motion estimates should be similar. Neither divergence profile displays Dines' compensation. The vertical motion profiles above 500 mb exhibit large, unrealistic values similar to those noted by Kurihara (1960) and Pfeffer (1962).

The standard deviation of the vertical motion random-error component estimated for the uppermost value of the dashed profile in figure 4B is 2.1×10^{-3} mb/s. This is an order of magnitude less than the computed value of -3.5×10^{-2} mb/s and indicates that the unrealistic result must be due to bias errors accumulated in the vertical integration.

In the example shown in figure 3, our conclusion was that polynominal filtering is able to remove sufficient random-error variance to enable filtered profiles to closely approximate true atmospheric profiles. Now a distinction must be made between the true profile and the expected profile. If all modeling assumptions were satisfied, then the expected profile would be identical to the true profile. When assumptions are violated, the expected profile will diverge from the true profile due to presence of bias errors. Two possible sources of bias error in this Bellamy technique are violation of the assumptions that (1) an unbiased filtering polynominal was selected in vertical filtering or (2) the wind field is linear within the area bounded by the triangle. The likelihood that the first assumption is satisfied was illustrated in figure 1. The possibility that the second assumption is violated in certain instances will now be considered.

The presence of nonlinearity in the wind field does not affect the property of filtering polynomials to reduce the effects of observational random-error components. However, if the wind field possesses nonlinear variation in the triangular region, then estimates of divergence by means of the Bellamy method may contain a bias error.

To illustrate this condition, we let the eastward component of a true, nonlinear wind field in a local region be given by the second-order Taylor series expansion,

$$u(x,y)=u_0+\left(\frac{\partial u}{\partial x}\right)_0(x-x_0)+\left(\frac{\partial u}{\partial y}\right)_0(y-y_0)+\frac{1}{2}\left(\frac{\partial^2 u}{\partial x^2}\right)_0(x-x_0)^2+\left(\frac{\partial^2 u}{\partial x\partial y}\right)_0(x-x_0)(y-y_0)+\frac{1}{2}\left(\frac{\partial^2 u}{\partial y^2}\right)_0(y-y_0)^2. \quad (7)$$

For simplicity, let u_0, $(\partial u/\partial y)_0$, $(\partial^2 u/\partial x^2)_0$, and $(\partial^2 u/\partial y^2)_0$ be zero and $(\partial u/\partial x)_0$ be equal to 3.0×10^{-5} s^{-1}. Because there usually is significant lateral shear variation in the horizontal, let $(\partial^2 u/\partial x\partial y)_0$ be equal to 3×10^{-10} $s^{-1}\cdot m^{-1}$. In figure 5, four triangles denoted by A, B, C, and D are located so that their vertexes lie on a circle with radius of 100 km centered at a common origin. The Bellamy divergence estimates at the single, common centroid computed using the true, discrete winds at the apex of the four triangles are 1.5×10^{-5} s^{-1} for A, 4.5×10^{-5} s^{-1} for B, and 3.0×10^{-5} s^{-1} for C and D. Clearly, two estimates, A and B, are biased while two, C and D, are unbiased even in the presence of the nonlinear field. The geometrical reason for the bias errors is associated with the orientation of the triangle in the nonlinear field and truncation error. Triangles C and D are laterally symmetric with respect to the x-axis while triangles A and B are not. In estimates from triangle B, divergence information comes from discrete winds along a line of y equal to +50 km where $\partial u/\partial x$ is larger than $(\partial u/\partial x)_0$, but the estimates are assigned to the common origin. For triangle A, information comes from the line y equal to −50 km where $\partial u/\partial x$ is small but is again assigned to the origin. In the triangles C and D, the lateral symmetric property averages the tendency to overestimate and underestimate divergence from discrete wind information above and below the x-axis. Thus, in a nonlinear wind field, Bellamy divergence estimates for the centroid are not unique but depend on orientation of the triangle with respect to the varying wind shear field.

Bias divergence errors must be present in either one or both of the contradictory profiles in figure 4A. Inspection of the 500-mb wind field suggests that the cause of the bias error component is likely associated with the opposite orientation of the Dayton–Washington–Greensboro and Nashville–Pittsburgh–Norfolk triangles relative to the nonlinear wind field.

Certainly, higher order wind variations exist in the atmosphere. For example, $\partial^2 u/\partial x\partial y$, $\partial^2 y/\partial x\partial y$, $\partial^2 u/\partial y^2$, and $\partial^2 v/\partial x^2$ are elements in horizontal advection of vorticity that tend to be extrema in regions of significant vertical motion. In the atmosphere, such nonlinear fields are maintained in the vertical through the thermal wind relation, and thus the bias error accumulates in the integration to estimate vertical motion. We conclude that significant bias errors exist in Bellamy divergence estimates and that bias errors will likely be largest in regions where the gradient of vorticity and vorticity advection is a maximum.

Divergence and Vertical Motion from Cross-Product and Quadratic Models

The methods of Bellamy (1949) and Endlich and Clark (1963) are special cases of the general polynomial model. In their methods, the number of coefficients to be estimated is equal to the number of observations, and an exact fit is always determined for the horizontal wind field. Two undesirable results of this condition are that no degrees of freedom are retained to study the "lack of fit" error for both bias and random components and that no horizontal filtering is accomplished. In the general model for which five and eight stations are used to estimate horizontal wind fields, these undesirable features are avoided. However, in using more stations it is necessary to increase the size of the local region. The a priori problem is whether or not the expansion of the local region can be kept small enough to permit an unbiased approximating polynomial.

TABLE 1.—*Overlapping triangles in local region used for Bellamy technique*

T1	New York, Greensboro, Pittsburgh
T2	Cape Hatteras, Greensboro, Washington
T3	New York, Cape Hatteras, Greensboro
T4	New York, Pittsburgh, Washington
T5	Greensboro, Pittsburgh, Washington
T6	Cape Hatteras, Greensboro, Pittsburgh
T7	New York, Cape Hatteras, Washington
T8	New York, Cape Hatteras, Pittsburgh

Our first consideration was the selection of a network arranged so that station location would constitute the basis of a desirable statistical design (Box and Wilson 1951, Box and Hunter 1957) while the local region of expansion would remain as small as possible. A region with five stations (Pittsburgh, New York, N.Y., Washington, Greensboro and Cape Hatteras, N.C.) arranged in quasi-rectangular fashion about one center station was selected. When postulating that the highest variation is quadratic in the local region, the quasi-rectangular array insures unbiased estimates in that the estimates of $(\partial u/\partial x)_0$ and $(\partial u/\partial y)_0$ are independent of $(\partial^2 u/\partial x^2)_0$ and $(\partial^2 u/\partial y^2)_0$. The center station provides one degree of freedom for filtering. If the station array formed a perfect rectangular array, the estimate of $(\partial u/\partial x)_0$ would also be independent of $(\partial^2 u/\partial x\partial y)_0$. A similar model is also valid for estimating $(\partial v/\partial y)_0$.

When quadratic variations are present, estimates of v_0 by the cross-product model will be biased. However, because the v_0 term in the divergence equation is usually an order of magnitude less than the divergence, it is likely that v_0 bias error does not contribute significantly to any vertical-motion bias component. Later, three additional stations, Buffalo, N.Y.; Albany, N.Y.; and Norfolk, Va. were added to form an expanded eight-station region and a full quadratic model is employed to eliminate the possibility of a bias error in v_0.

After the five-station array was selected, preliminary checks were made to (1) ascertain whether the nonlinear wind field evident in the results of the overlapping triangles extended downstream through the five-station region and (2) test the assumption of the unbiased vertical filtering. In the first check, Bellamy divergence and vertical-motion estimates were made for eight different triangle combinations in the five-station region. Station vertexes for eight triangles identified by a letter and number are listed in table 1 and the vertical motion profiles with corresponding identification are presented in figure 6.

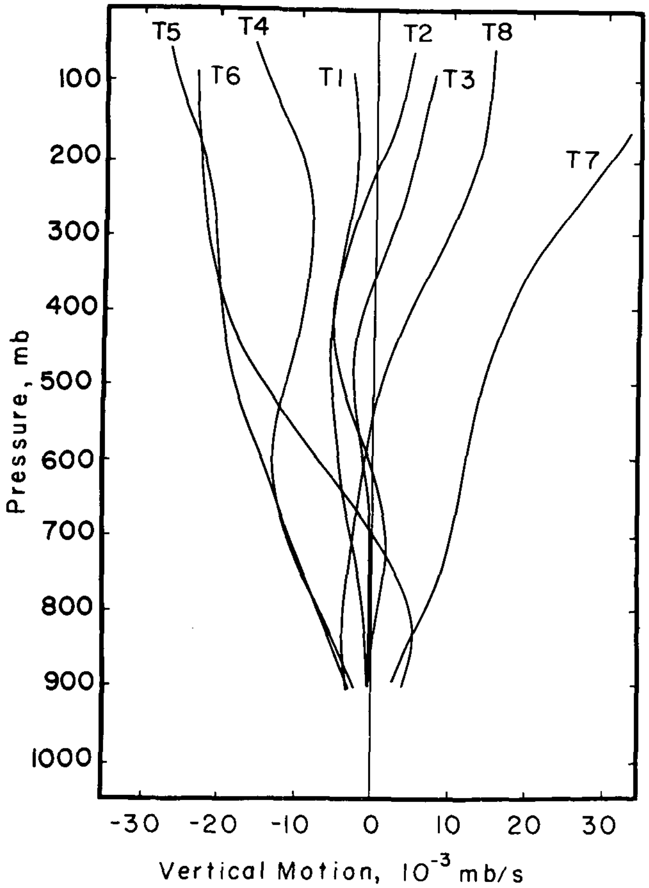

FIGURE 6.—Profiles of vertical motion by the Bellamy method for eight different triangles in the eastern United States for 0000 GMT, Dec. 12, 1960. Triangle identification is given at the profile top and listed in table 1.

TABLE 2.—*Twelve-day mean rms wind vector error estimates from Cape Hatteras, Greensboro, Washington, Pittsburgh, and New York*

Level (mb)	Estimate from lack of fit (m/s)	Ellsaesser's estimates (m/s)
700	2. 76	2. 42
500	3. 14	2. 88
300	4. 98	5. 16
200	5. 29	5. 70

The contradictory results indicate failure of the Bellamy model to portray reasonable synoptic similarity. Thus, the nonlinear wind field in the local region provides data suited to study estimation by higher order models.

The second check consisted of both an inspection and a random-error variance analysis of the filtered vertical-wind profiles. No significant bias errors appeared to be present. Twelve-day mean root-mean-square (rms) vector wind error estimates for five stations are presented and compared with Ellsaesser's (1957) results in table 2. The similarity achieved by averaging 60 profiles is excellent.

Profiles of divergence and vertical motion computed from four different models for Dec. 12, 1960, are presented in figure 7. A linear model using five stations (L5), a cross-product model using five and eight stations (C5, C8) and a quadratic model using eight stations (Q8) were used for

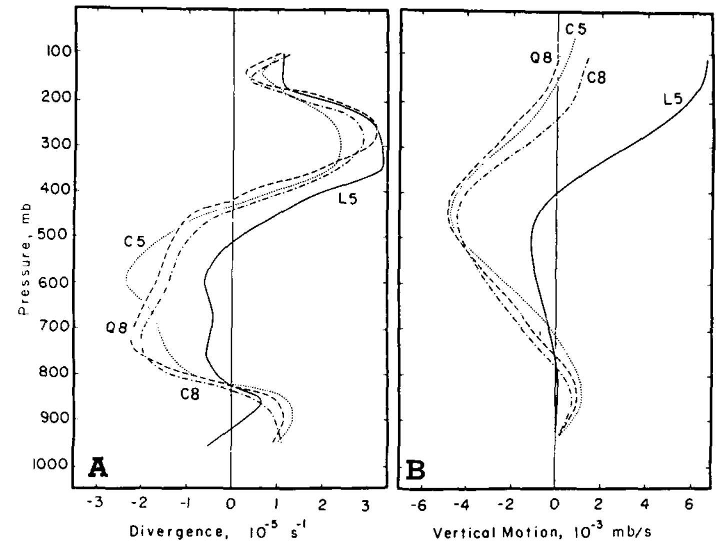

FIGURE 7.—Filtered (A) divergence and (B) vertical-motion profiles for the origin of the approximating polynominal region in the eastern United States for 0000 GMT, Dec. 12, 1960.

TABLE 3.—*Standard deivation estimates for divergence (Div, 10^{-5} s^{-1}) and vertical motion (ω, 10^{-3} mb/s) for 0000 GMT, Dec. 12, 1960*

Level (mb)	L3		L5		C5		C8	
	Div	ω	Div	ω	Div	ω	Div	ω
850	0. 75	0. 69	0. 61	0. 63	0. 03	0. 61	0. 42	0. 38
700	0. 67	1. 40	0. 57	1. 12	0. 64	1. 24	0. 42	0. 78
500	0. 63	1. 67	0. 43	1. 33	0. 49	1. 48	0. 37	0. 96
300	1. 16	2. 00	0. 76	1. 55	0. 87	1. 73	0. 67	1. 17
200	1. 10	2. 45	0. 82	1. 76	0. 94	1. 98	0. 70	1. 39
100			0. 93	1. 90	1. 06	2. 14	0. 94	1. 59
50								

comparison. All vertical-motion profiles were computed by setting ω(950 mb) equal to zero to facilitate comparison of the different models. Because ω(950 mb) is usually small compared to midtropospheric values, the error introduced by this simplification is minor.

The four profiles in figure 7A indicate low-level divergence, midtropospheric convergence, and upper level divergence. The low-level divergence appears reasonable because it is associated with anticyclonic isobaric curvature in the local region (fig. 2). Figure 7B shows that stratospheric estimates of ω from the higher order models tend to zero, a feature which suggests superior results. From table 3, we see that the one-standard-deviation estimates of ω at 100 mb for the C5 and C8 models are 2.14×10^{-3} and 1.59×10^{-3} mb/s. Thus, their actual departure from zero for this situation may be entirely explained by a random-error component. The L5 model produced less acceptable results. A departure of 6.4×10^{-3} mb/s from zero at the 100-mb level for the L5 model exceeds a three-standard-deviation estimate (table 3) and is almost certainly too large to be explained by the random-error component. However, the uppermost values of ω in figure 7 are not nearly as great as those indicated for the eight Bellamy profiles in figure 6.

In these results, we determine whether or not the uppermost value of ω is biased from estimates of the vertical motion random-error variance. The standard deviation estimates for divergence and vertical motion presented in table 3 were computed from estimates of random-error variance from the initial vertical filtering. The tendency for the divergence standard-deviation estimates to increase with height is due to the increase of the wind variance with height. The increase in standard-deviation estimates of vertical motion is due to this effect and the accumulation of a random-error component through vertical integration of divergence. Comparing the models, we see that the random-error variance tends to be reduced as the local region is expanded and the degrees of freedom for horizontal filtering (i.e., the number of stations minus the polynomial coefficients) are increased.

In figure 7, one cannot conclusively state which divergence or vertical-motion profile is closest to the true value. However, in the midtroposphere, the systematic difference of divergence estimates between the linear and higher order models is large enough to indicate the presence of higher order variation in the wind field that was evident in the contradictory results from the Bellamy method. These results tentatively support the use of higher order polynomial models in preprocessing wind fields used for estimating divergence and vertical motion.

When computing divergence only, one uses estimates of linear derivatives of the wind field. Consequently, when selecting a model one need not be concerned with estimating the entire set of higher order polynomial coefficients, but rather with choosing a model that will produce unbiased estimates of the linear derivatives. Because the order of the selected model is truncated by necessity (due to the marginal rawinsonde station density), the design of the model (Box and Wilson 1951, Box and Hunter 1957), that is, the location of the reporting stations within the local region, determines to what extent the linear derivatives may be biased by the various orders of variation in the wind field. In these situations, selection of an unbiased model is extremely complicated. There is no way of knowing the order of variation a priori and of insuring an optimum selection of a higher order model for a particular situation.

Another possible source of the bias error is the violation of the assumption that winds are observed directly above the observing station. In reality, high-altitude observations are measurements made at an appreciable distance downstream. If lateral wind shears are present, the relative positions of the ballon will change with time causing biased divergence estimates. Although this bias is usually small (Kurihara 1960), the deformation of relative balloon positions is appreciable in the vicinity of a jet core. When this systematic divergence error is integrated, vertical-motion estimates must be biased.

Temporal Continuity of Vertical-Motion Estimates

To portray temporal continuity, we will compare the cross-product model with the Bellamy method for a synoptic situation in which the Bellamy method produces reasonable estimates. The best situation, in which winds appeared to vary linearly in the local region, occurred between 1200 GMT on Dec. 2 and 1200 GMT on Dec. 3, 1960. During this period, the surface high-pressure system and upper air ridge remained relatively stationary. The cross-product model was applied for the five-station region and the Bellamy model for the eight triangles listed in table 3. The results from T2, which is a nearly equilateral triangle bounded by Cape Hatteras, Greensboro, and Washington in the lower half of the five-station region, were selected for comparison because it was the only triangle that produced reasonable estimates for each of the 3 synoptic hr.

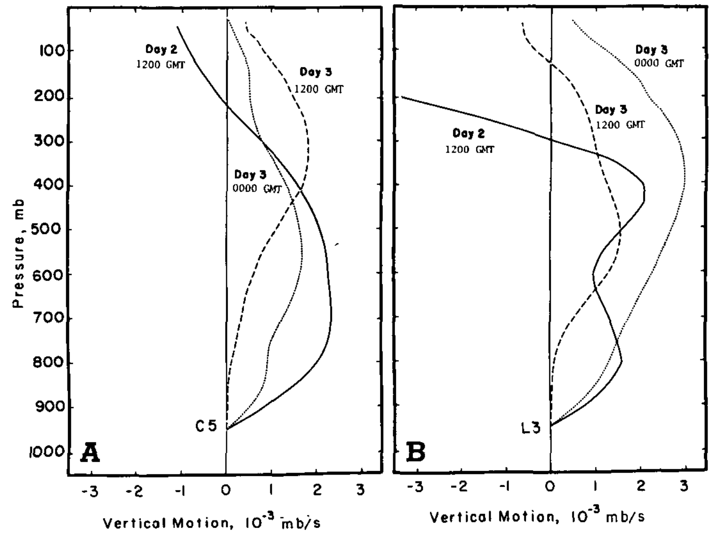

FIGURE 8.—Filtered vertical motion by (A) the cross-product model and (B) the Bellamy method for triangle bounded by Cape Hatteras, N.C.; Greensboro, and Washington, D.C., from 1200 GMT, Dec. 2 through 1200 GMT, Dec. 3, 1960.

Successive profiles of vertical motion by the two models are presented in figure 8. In a detailed comparison, all three profiles from the cross-product model satisfy upper boundary conditions in a more convincing fashion than the Bellamy profiles. Vertical-motion profiles display an upward shift of the region of maximum downward motion as the upper air high-pressure ridge moves slowly eastward from western Pennsylvania into the local region. However, this feature is more prominent and shows better time continuity in the cross-product profiles. Because the cross-product model results were compared with the best of eight different results from the Bellamy method, we conclude that, even in situations where the Bellamy model produces valid estimates, the cross-product estimates are better.

4. CONCLUSIONS AND SUGGESTIONS FOR FUTURE RESEARCH

The problem of unbiased interpolation of fields from discrete measurements is extremely difficult if not impossible when the density of the reporting stations is marginal. The positive aspect of this research is that the problem of "spurious kinematic vertical motion" in the upper atmosphere has been isolated and alternative techniques proposed.

The results show that vertical filtering of wind observations is desirable to suppress effects of random errors in estimating divergence and vertical-motion profiles and to gain estimates of the random-error variance. An increased sampling rate to provide more vertical observations of the balloon positions is also desirable because, as noted by Rachele and Duncan (1967), the profiles would be more representative of the true wind. With an increased sampling rate, approximating polynomial estimates would be even more accurate due to the central limit effect in reducing the random-error variance component.

The most serious problem in kinematic vertical-motion calculations lies in estimation of the horizontal divergence, which belongs to the more general problem of two-dimensional interpolation. It is emphasized that (1) utilization of diagnostic techniques that yield unbiased estimates of atmospheric variables and (2) incorporation of a means of checking the validity of modeling assumptions are extremely important considerations. Our results indicate that estimates of horizontal divergence and vertical motion from wind gradients computed with cross-product and quadratic polynomial models are better than those obtained from the linear model. Bias errors present in Bellamy estimates of divergence have been traced to violation of the modeling assumption of linear wind variation. Regions of nonlinear wind variation in many instances are also regions where the gradients of vorticity, vorticity advection, and vertical motion are extrema. Thus, the assumption of linearity utilized in the Bellamy technique is inconsistent with the expected velocity distributions where more active weather situations are occurring.

An alternate method would be to objectively analyze data from the rawinsonde network to a regular grid and then utilize conventional finite-differencing operations that are valid through either second or fourth order to estimate horizontal divergence. Such a procedure is consistent with the idea of using higher order approximating polynomials to gain unbiased estimates of the first-order derivatives. However, this technique would produce unbiased estimates only if the rawinsonde winds were interpolated to a regular grid by an approximating function that produced unbiased wind component estimates. Due to the present irregular rawinsonde network, it is unlikely that any objective analysis technique exists that will produce unbiased estimates at fixed gridpoints under all possible configurations of atmospheric variables for the synoptic scale.

From considerations of typical wind distributions in quasi-geostrophic balance, optimum statistical designs, and our diagnostic results, the cross-product model is the lowest order model that should be used for direct kinematic vertical motion estimates using the present United States rawinsonde network. Even though higher order modeling tends to reduce biased estimates of vertical motion, the cross-product and quadratic results for other situations presented by Schmidt and Johnson (1969) indicate that bias errors are not always eliminated. In an ideal situation, the choice of model should be based on the principal features of the wind field and the pattern of reporting stations in each local area. The danger in selecting a unique order for the interpolating polynomial is that a single model is not optimized to produce unbiased estimates in all situations. To reduce the polynomial order required for unbiased estimation for wind fields associated with asymmetric ridges and intense troughs or jets, we should attempt to gain symmetry of the wind field with respect to both the origin and axes of the scaled independent variables through translation and rotation of the independent variables. Such techniques will improve the statistical design for the particular wind field and thereby reduce the tendency for biased estimates.

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REFERENCES

Bellamy, John Carey, "Objective Calculations of Divergence, Vertical Velocity and Vorticity," *Bulletin of the American Meteorological Society*, Vol. 30, No. 2, Feb. **1949**, pp. 45–49.

Box, G. E. P., and Hunter, J. S., "Multi-Factor Experimental Designs for Exploring Response Surfaces," *Annals of Mathematical Statistics*, Vol. 28, No. 1, Baltimore, Md., **1957**, pp. 195–241.

Box, G. E. P., and Wilson, K. B., "On the Experimental Attainment of Optimum Conditions," *Journal of the Royal Statistical Society*, Ser. B, Vol. 13, No. 1, London, England, **1951**, pp. 1–45.

Danielsen, Edwin F., "Research in Four-Dimensional Diagnosis of Cyclonic Storm Cloud Systems," *Scientific Report* No. 1, Contract No. AF 19(628)-4762, Department of Meteorology, Pennsylvania State University, University Park, Jan. **1966**, 53 pp.

De Mandel, R. E., and Scoggins, James R., "Mesoscale Wave Motions as Revealed by Improved Wind Profile Measurements," *Journal of Applied Meteorology*, Vol. 6, No. 4, Aug. **1967**, pp. 617–620.

Eddy, Amos, "The Objective Analysis of Horizontal Wind Divergence Fields," *Quarterly Journal of the Royal Meteorological Society*, Vol. 90, No. 386, London, England, Oct. **1964**, pp. 424–440.

Ellsaesser, Hugh W., "Errors in Upper-Level Wind Forecasts," *Bulletin of the American Meteorological Society*, Vol. 38, No. 9, Nov. **1957**, pp. 511–517.

Endlich, R. M., and Clark, J. R., "Objective Computation of Some Meteorological Quantities," *Journal of Applied Meteorology*, Vol. 2, No. 1, Feb. **1963**, pp. 66–81.

Gilchrist, Bruce, and Cressman, G. P., "An Experiment in Objective Analysis," *Tellus*, Vol. 6, No. 4, Stockholm, Sweden, Nov. **1954**, pp. 309–318.

Hildebrand, Francis B., *Introduction to Numerical Analysis*, McGraw-Hill Book Co., Inc., New York, N.Y., **1956**, 511 pp.

Johnson, D. H., "Preliminary Research in Objective Analysis," *Tellus*, Vol. 9, No. 3, Stockholm, Sweden, Aug. **1957**, pp. 316–322.

Johnson, D. R., "The Role of Terrestrial Radiation in the Generation of Available Potential Energy," Ph. D. thesis, Department of Meteorology, University of Wisconsin, Madison, Jan. **1965**, 121 pp.

Kendall, Maurice G., and Stuart, A., *The Advanced Theory of Statistics: Vol. 1. Distribution Theory*, Hafner Publishing Co., New York, N.Y. **1958**, 457 pp.

Kendall, Maurice G., and Stuart, A., *The Advanced Theory of Statistics: Vol. 2. Statistical Inference and its Statistical Relationship*, Hafner Publishing Co., New York, N.Y., **1961**, 521 pp.

Krishnamurti, T. N., "A Study of a Developing Wave Cyclone," *Monthly Weather Review*, Vol. 96, No. 4, Apr. **1968**, pp. 208–217.

Kurihara, Y., "Numerical Analysis of Atmospheric Motions," *Journal of the Meteorological Society of Japan*, Ser. 2, Vol. 38, No. 6, Tokyo, Dec. **1960**, pp 288–304.

Palmén, Erik Herbert, "Vertical Circulation and Release of Kinetic Energy During the Development of Hurricane Hazel Into an Extratropical Storm," *Tellus*, Vol. 10, No. 1, Stockholm, Sweden, Feb. **1958**, pp. 1–23.

Panofsky, Hans A., "Objective Weather-Map Analysis," *Journal of Meteorology*, Vol. 6, No. 6, Dec. **1949**, pp. 386–392.

Perkins, Donald Theodore, "The Response of Balloons to the Wind," *Bulletin of the American Meteorological Society*, Vol. 33, No. 4, Apr. **1952**, pp. 135–139.

Petterssen, Sverre, *Weather Analysis and Forecasting*, 2d Ed., 2 volumes, McGraw-Hill Book Co., Inc., New York, N.Y., **1956**, 428 pp. and 266 pp.

Pfeffer, Richard L., "Results of Recent Research in Meteorology at the Lamont Geological Observatory," *Proceedings of the International Symposium on Numerical Weather Prediction, Tokyo, Japan, November 7–13, 1960*, Meteorological Society of Japan, Tokyo, Mar. **1962**, pp. 249–263.

Rachele, Henry, and Duncan, Louis D., "Desirability of Using a Fast Sampling Rate for Computing Wind Velocity From Pilot-Balloon Data," *Monthly Weather Review*, Vol. 95, No. 4, Apr. **1967**, pp. 198–202.

Reiter, Elmar R., *Jet-Stream Meteorology*, University of Chicago Press, Ill., **1963**, 515 pp.

Schmidt, P. J., and Johnson, D. R., "The Use of Approximating Polynomials to Estimate Profiles of Winds, Divergence and Vertical Motion," *Annual Report*, Contract No. E-8-69(G), Department of Meteorology, University of Wisconsin, Madison, Dec. **1969**, 122 pp.

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